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# RHEOLOGICAL FACTOR IN THE THERMAL PROBLEM OF SHF-HYPERTHERMIA-TREATMENT. 2. SIMULATION OF RELATIVE BLOOD FLOW VARIATION

**Z. P. Shul'man and A. A. Makhanyok**

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*The rheological effect on the temperature field in a homogeneous tissue under SHF hyperthermia treatment is discussed.*

We consider the influence of the Il'yushin parameter

$$\mathrm{Il} = \frac{\tau_0 D}{\mu_p \bar{u}} \quad (1)$$

on tissue heating at some fixed values of the thermophysical parameters with use of different blood flow models.

Blood as a suspension of its cells (leukocytes, erythrocytes, etc.) in plasma has a substantial non-Newtonian character. The rheological behavior of a nonlinear-viscoplastic medium in the general case is described by the Shul'man equation [1]

$$\tau^{1/n} = \tau_0^{1/n} + (\mu_p \gamma)^{1/m}. \quad (2)$$

At present the special case of Eq. (2) for $n = m = 2$, namely, the Casson model is most commonly used. In [2], it is proposed to subdivide the curve of normal blood flow into three sections: 1) Casson flow ($\dot{\gamma} = 0.5$-$25$ sec$^{-1}$), 2) Newtonian flow ($\dot{\gamma} > 100$ sec$^{-1}$), and 3) intermediate flow between above.

Until recently the Casson flow curve has been assumed valid for normal blood when the latter is not subjected to various (thermal, chemical, radiative) effects. A deviation from the Casson dependence is assumed in the case of pathologies (ischemia, diabetes, blood diseases, etc.). However, numerous bacteriological analyses conducted at the town of Nizhnii Novgorod in recent years have revealed that the Casson flow curve (CFC) is not adequate for the behavior of normal blood. Figure 1 shows the results of rheometry experiments conducted and processed by A. N. Sundukov. He has used three pairs of nonlinear viscoplasticity parameters in the general law (2): a) $n = m = 1$, the Schvedov-Bingham fluid (a linear model); b) $n = m = 2$, the Casson fluid flow; c) $n = m = 3$, a more general relation.

Inadequacy of the CFC is especially evident for small ($\dot{\gamma} < 0.1$ sec$^{-1}$) and very small ($\dot{\gamma} < 0.01$ sec$^{-1}$) shear rates. But it is just this range where the most important deaggregation-aggregation processes of structural formations of blood cells take place. Processing in terms of $\tau^{1/2} - \dot{\gamma}^{1/2}$ does not linearize the initial section of the experimental flow curve, which, in turn, leads to incorrect prediction of the an important rheological parameter of blood, namely, yield stress $\tau$. But processing in terms of $\tau_0^{1/3} - \dot{\gamma}^{1/3}$ results in practically complete linearization of the actual flow curve of normal blood. Each pathology, in fact, must reflect on the rheological behavior of blood, which is indirectly expressed through a change of the ESR. We may say with assurance that the flow curve must be characterized for many diseases by $n \neq m$ within $1 \leq n \leq 3$; $1 \leq m \leq 3$, with $n$ and $m$ not necessarily being integers.

For any inelastic non-Newtonian fluid flow in a cylindrical pipe its mass flow rate $Q$ and flow curve $f(\tau)$ are related by the universal Mooney-Rabinovich equation [1]

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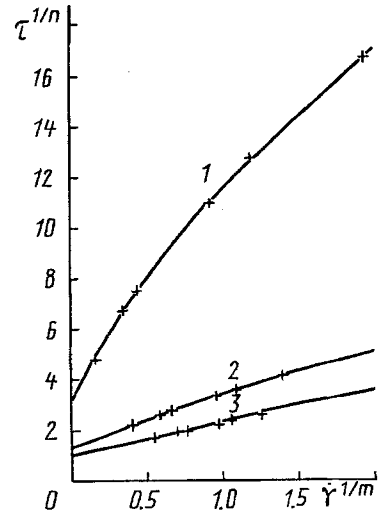



Fig. 1. Data of A. N. Sundukov on blood rheology (healthy persons at age 30): 1) $n = m = 1$; 2) 2; 3) 3. $\tau^{1/n}$, $(\mathrm{mPa})^{1/n}$; $\dot{\gamma}^{1/m}$, $\sec^{-1/m}$.

$$\frac{Q}{\pi R^3} = \frac{1}{\tau_w^3} \int_{\tau_0}^{\tau_w} \tau^2 f(\tau)\, d\tau\,, \tag{3}$$

where $R$ is the pipe radius; $\tau$ is the shear stress on its wall.

In our cases

$$f(\tau) = (\tau^{1/n} - \tau_0^{1/n})^m\,, \tag{4}$$

where $n = m = 3$ or $n = m = 2$. Substituting (4) into (3) and integrating, we arrive at the following expressions for the relative flow rates:

$$Q_{n,m}/Q_H = \varphi_{n,m}(\zeta)\,, \tag{5}$$

where

$$\varphi_{n=m=2}(\zeta) = 1 - \frac{16}{7}\zeta^{1/2} + \frac{4}{3}\zeta - \frac{1}{21}\zeta^4\,, \tag{6}$$

$$\varphi_{n,m=3}(\zeta) = 1 - \frac{36}{11}\zeta^{1/3} + \frac{36}{10}\zeta^{2/3} - \frac{4}{3}\zeta + \frac{1}{165}\zeta^4\,. \tag{7}$$

Here $\zeta = \tau_0/\tau_w = l_0/R$ is the relative width of the quasisolid core; $Q_H = \pi R^4 \Delta P / 8\mu_p L$ is the Newtonian (Poiseuille) fluid flow rate through the cross-section of a cylindrical pipe.

Equation (5) gives the change in the flow rate relative to the Newtonian analog. In the general case, it does not correspond to the relative change in blood flow (perfusion $f$ or mass velocity of blood flow $w_b$) before and after hyperthermia treatment since before this procedure the blood is non-Newtonian and is characterized in part by the viscoplasticity parameter $\mathrm{Il}_{in}$.

Because of the complicated nature of blood suspension the Il'yushin parameter fails to adequately describe the rheological behavior of blood. The non-Newtonian character of blood is manifested also in the absence of structurization, i.e., at $\tau_0 = 0$. In this case, pseudoplasticity is retained, i.e., viscosity decreases with increase in the shear rate, and the blood behaves like a nonlinear-viscous nonplastic fluid.

A simultaneous and more complete account of nonlinear viscosity and viscoplasticity is attained when $n$ and $m$ are not integers or are not equal to each other. As a rule, these parameters are not integers in practice, even if they are close in value. However, $\tau_0$ as the structural characteristic of blood is the governing factor both in normality and pathology. Therefore in our analysis we prefer to use the Il'yushin parameter for the present.

In the general case $n \neq m$ the nonlinear viscosity is characterized by the complex [3]:

$$S = \frac{\tau_0 L^{n/m}}{\mu_p^{n/m} \bar{u}^{n/m}}, \tag{8}$$

which turns into the parameter Il at $n = m$.

From definition (1) of the Il'yushin parameter and equation (3) we can easily derive an expression for the relative change in the fluid flow rate through a cylindrical channel for arbitrary positive $n$ and $m$. If $n$ and $m$ are integers, we may write

$$f(\tau) = \frac{1}{\mu_p}(\tau^{1/n} - \tau_0^{1/n})^m = \frac{1}{\mu_p}\sum_{k=0}^{m}(-1)^k C_m^k \tau^{(m-k)/n} \tau_0^{k/n}.$$

Substituting this equation into (3), we obtain (see [1])

$$Q = \frac{n}{3n+m}\frac{\pi R^3}{\mu_p}\tau_w^{m/n}\varphi_{n,m}(\zeta), \tag{9}$$

where

$$\varphi_{n,m}(\zeta) = \sum_{k=0}^{m}(B_k/B_0)(\zeta^{k/n} - \zeta^{m/n+3}), \quad B_k = \frac{(-1)^k C_m^k}{3 + (m-k)/n}. \tag{10}$$

If $n$ and $m$ are the rational numbers, then expanding $f(\tau)$ into a series in $\tau$ and substituting it in (3), we arrive at an equation for a flow rate, similar to (9) (see [1]), in which

$$\varphi_{n,m}(\zeta) = \sum_{k=0}^{\infty}(B_k/B_0)(\zeta^{k/n} - \zeta^{m/n+3}).$$

It is pertinent to note that $\varphi_{n,m}(\tau_0 = 0) = 1$, $\varphi_{n,m}(\tau_w = \tau_0) = 0$.

From the definition of (8) and equation (9) we have

$$S = \frac{\tau_0 (2R)^{n/m}}{\mu_p^{n/m}(Q/\pi R^2)^{n/m}} = \left(2\left(3 + \frac{m}{n}\right)\right)^{n/m}\frac{\zeta}{\varphi_{n,m}^{n/m}(\zeta)}. \tag{11}$$

For the relative change in flow rate we may write

$$\frac{Q}{Q_{in}} = \left(\frac{r}{R_{in}}\right)^3 \frac{\mu_{p\,in}}{\mu_p}\left(\frac{\tau_w/\tau_0}{\tau_{w\,in}/\tau_{0\,in}}\right)^{m/n}\left(\frac{\tau_0}{\tau_{0\,in}}\right)^{m/n}\frac{\varphi_{n,m}(\zeta)}{\varphi_{n,m}(\zeta_{in})} =$$

$$= \left(\frac{R}{R_{in}}\right)^3 \frac{\mu_{p\,in}}{\mu_p}\left(\frac{\zeta_{in}}{\zeta}\right)^{m/n}\left(\frac{\tau_0}{\tau_{0\,in}}\right)^{m/n}\left(\frac{\zeta}{\zeta_{in}}\right)^{m/n}\left(\frac{S_{in}}{S}\right)^{m/n}$$

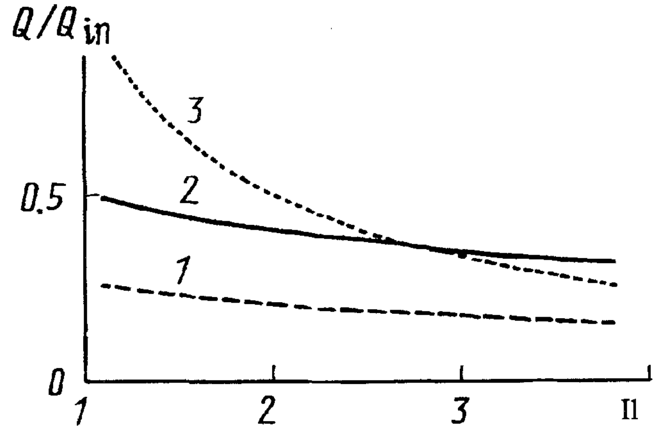
and, consequently,

$$Q/Q_{in} = (R/R_{in})^3(\mu_{p\,in}/\mu_p)(\tau_0 S_{in}/\tau_{0\,in} S)^{m/n}. \tag{12}$$

In deriving Eq. (12) we have assumed that $n$ and $m$ remain unchanged.

In the special case $n = m$, $S$ is transformed into the Il'yushin parameter Il whereas Eq. (10) is reduced to (6) for $n = m = 2$ and to (7) for $n = m = 3$. Next, we may describe a change in the fluid flow rate as

$$\frac{Q}{Q_{in}} = \left(\frac{R}{R_{in}}\right)^3 \frac{\mu_{p\,in}}{\mu_p}\frac{\tau_0}{\tau_{0\,in}}\frac{\mathrm{Il}_{in}}{\mathrm{Il}}. \tag{13}$$



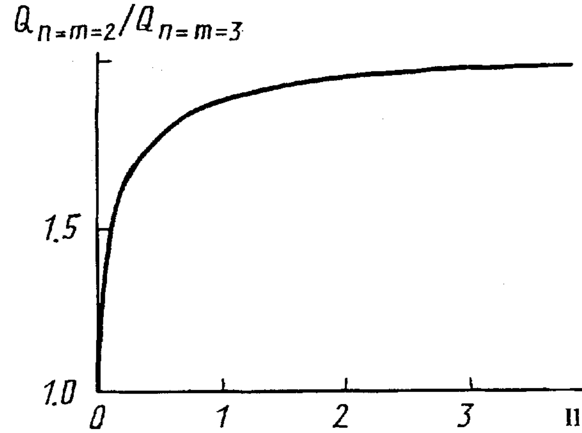




Fig. 2. Relative fluid flow rate for different flow models: 1) $n = m = 3$; 2) $n = m = 2$; 3) $R = R_{in}$ in (14).

Fig. 3. Flowrate ratio of a Casson fluid and a fluid with $n = m = 3$ in its flow equation.

If $\tau_0 = 0$, then from (13) we have $Q/Q_{in} = (R/R_{in})^{3+m/n}(\mu_{p_{in}}/\mu_p)$ (assuming that $\Delta p/L = \text{const}$).

Since at present we have no data on the change in the analog of plastic viscosity and yield stress during hyperthermia treatment, we assume that $\mu_p = \mu_{p_{in}}$ and $\tau_0 = \tau_{0_{in}}$. Then the relative change in the flow rate and, consequently, in the mass velocity of blood flow may be written in a first approximation as

$$\frac{w_b}{w_{b\,in}} = \left(\frac{R}{R_{in}}\right)^3 \frac{\mathrm{Il}_{in}}{\mathrm{Il}}. \tag{14}$$

Figure 2 shows the relative change in the blood flow rate calculated by equations (5)-(7) and by (14) for $R = R_{in}$ and $\mathrm{Il}_{in} = 1$. As is seen, the calculation by (14) for $R = R_{in}$ and $\mathrm{Il} > 2$ shows that the relative change in flow rate is close to the Casson case calculated by (6). For $\mathrm{Il} < 2$, the calculations by (14) produce much higher relative flow rates than when Eqs. (5)-(7) are used. In addition, whereas in the range $\mathrm{Il} = 1$-$4$ the flow rate of the fluids with $m = n = 2$ and $m = n = 3$, evaluated relative to the Newtonian fluid flow rate, undergoes an approximately twofold decrease, the flow rate calculated by (14) within the same Il'yushin parameter range shows a fourfold decrease. As is seen, the relative change in flow rate is most pronounced within the range of low viscoplasticity.

A comparison of flow rates for $n = m = 2$ and $n = m = 3$ is given in Fig. 3. It is noteworthy that $\lim_{\mathrm{Il}\to\infty} Q_{n=2}/Q_{n=3} = \infty$ rather than 2, as might seem from the figure.

According to (14), tissue perfusion changes proportionally to the change in size (diameter) of vessels to the third power. Consequently, equally with the Il'yushin parameter the ratio $(R/R_{in})$ determines the intensity of heat removal by blood from the heating zone under local hyperthermia. Many works report different data on the change in diameter of vessels exposed to different factors, including thermal action. For instance, in the case of local hyperthermia of melanoma A-Mel-3 vessel diameters change rather slightly in the temperature range from 30 to 42.5°C [4]. But the diameter of vessels of normal tissue may increase by a factor of 1.5 [5].

Figure 4a shows the calculated maximum tissue temperature as a function of the Il'yushin parameter using different methods for describing the mass velocity of blood flow (by equations (6), (7) and by (14) for $R = R_{in}$ and $R = 1.5R_{in}$). As the initial mass velocity of blood flow, we have adopted $w_{b\,in} = 9$ kg/(m$^3$·sec). The calculation is made at $P = 15$ W, $\lambda = 0.4$ W/(m·K), Bi = 12.

Figure 4 illustrates the importance of a proper choice of the method to describe blood flow (the difference in the maximum temperatures attains 15 deg). Whereas in the normal tissue for the chosen values of the thermophysical parameters and $R = 1.5R_{in}$ the temperatures are in the subcritical range, in tissues with $R = R_{in}$ they attain dangerous values (>45°C). Calculations for the Casson fluid and the change of blood flow according to (5), (6) give safe temperatures only for $\mathrm{Il} < 2$.

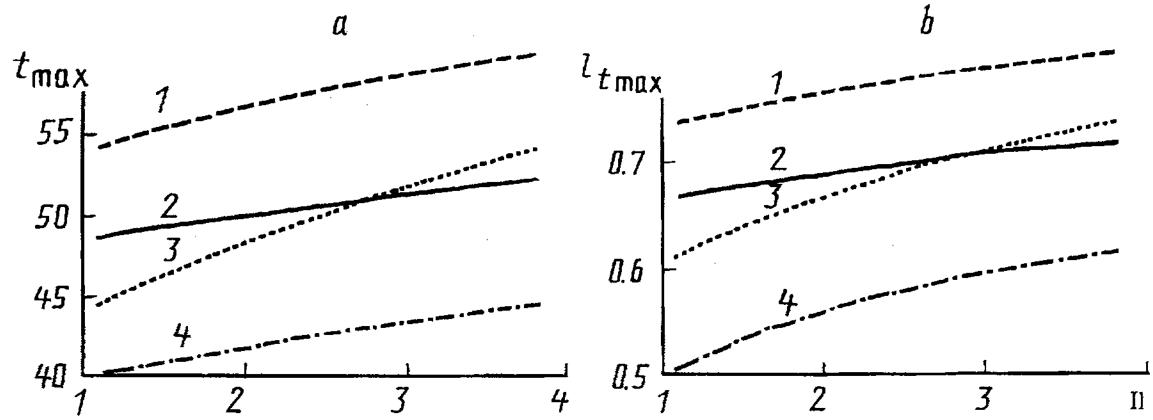



Fig. 4. Maximum tissue temperature (a) and localization depth of maximum tissue heating (b) as a function of the Il'yushin parameter: 1) $n = m = 3$; 2) $n = m = 2$; 3) $R = R_{in}$ in (14); 4) $R = 1.5R_{in}$ in (14). $t_{max}$, °C; $Il_{max}$, cm.

The change in the depth at which maximum heating is achieved is shown in Fig. 4b. A common tendency, independently of the blood flow model and the method for describing the change in perfusion, is increase in the depth of maximum heating of the tissue with increase in the Il'yushin parameter. However, on the whole this depth is rather small (<1 cm).

In conclusion, the results of numerical simulation of the change in blood flow as well as in heating of homogeneous tissue are indicative of the substantial rheological effect of blood flow on the hyperthermia process.

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## NOTATION

$\lambda$, thermal conductivity of the tissue; $f$, perfusion; $w_b$, mass velocity of blood flow; Bi, Biot number; $D$, capillary diameter; $R$, capillary radius; $\bar{u}$, average blood velocity in the capillary; $\tau_0$, yield stress; $\mu_p$, analog of plastic viscosity; $\dot{\gamma}$, shear rate; $\tau$, shear stress; $n$, $m$, parameters of the phenomenological Shul'man equation; $Q$, blood flow rate; $f(\tau)$, flow curve; $\tau_w$, shear stress on the capillary wall; $\zeta$, relative width of the quasisolid core; $\Delta p/L$, pressure gradient in the capillary; Il, rheological factor (Il'yushin parameter). Subscripts: in, initial value of a parameter (before heating).

## REFERENCES

1. B. M. Smol'skii, Z. P. Shul'man, and V. M. Gorislavets, Theodynamics and Heat Transfer of Nonlinear-Viscoplastic Materials [in Russian], Minsk (1970).
2. M. N. Firsov, Macro- and Microrheology of Blood in Normality and Pathology, Doctoral Dissertation in Biology, Moscow (1983).
3. Z. P. Shul'man, Problem of Heat and Mass Transfer [in Russian], Moscow (1970).
4. R. S. Reindols and B. Endrich, Int. J. Treatment Hyperthermia, **2**, No. 2, 111-137 (1986).
5. V. A. Levtov, S. A. Regirer, and N. Kh. Shadrina, Rheology of Blood [in Russian], Moscow (1982).